



Stochastic and Deterministic Fault Detection for Randomized Gossip Algorithms

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Objectives

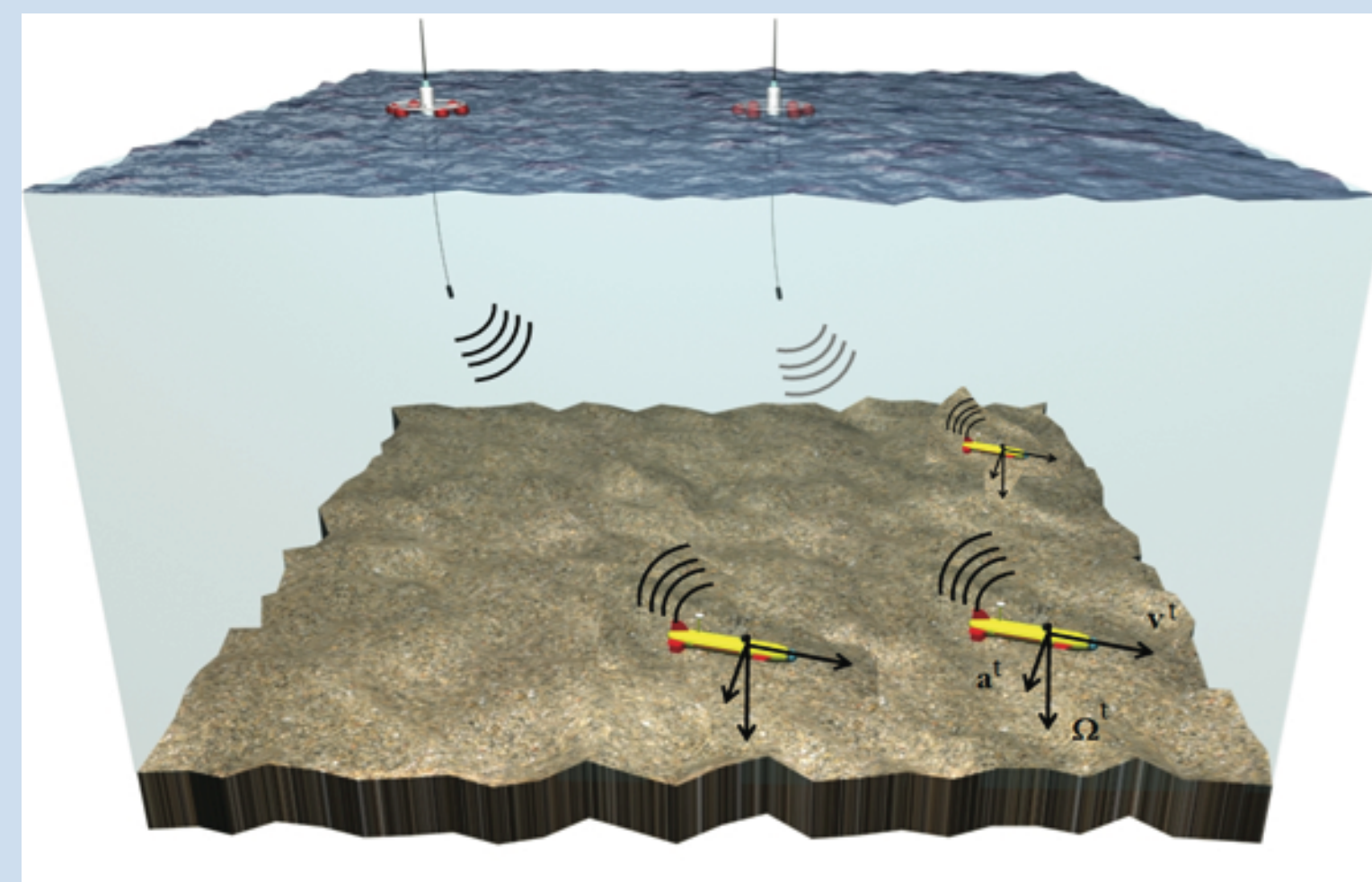
- Compute the average of the initial states in finite-time
- Guarantee fault detection and bounds on the maximum possible deviation from an attack
- Incorporate the transmissions stochastic information in the fault detection mechanism.

① Motivation

Nodes need to distributedly agree on a common value:

- Smart Grids when deciding the needed power
- Robot swarms to find rendezvous points
- Social Networks making a pool about a subject.

In all these examples a single node can drift the entire network to any desired value!



② Problem Statement

The objective is to compute the average of some quantity of interest. If faulty, any node must detect the fault using only local information communicated from their neighbors.

$$G = (V, E) \quad E \subseteq V \times V$$

$$S^i : \begin{cases} x(k+1) = \left(A_0 + \sum_{\ell=1}^{n_\Delta} \Delta_\ell(k) A_\ell \right) x(k) + B(k)u(k) \\ y^i(k) = C^i(k)x(k) \end{cases}$$

n_Δ number of uncertainties

Each S^i is a Linear Parameter-Varying (LPV) system

$\Delta_\ell(k)$ are scalar uncertainties

Byzantine Consensus Problem:

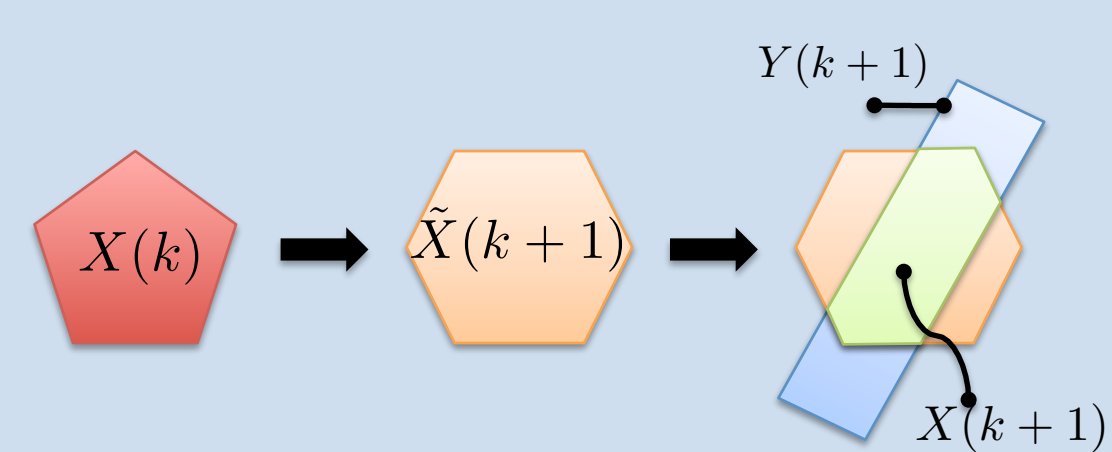
Either detect non-zero signals $u(k)$ using $y(k)$ without the knowledge of $B(k)$ or compute the final consensus value.

③ Proposed Solution

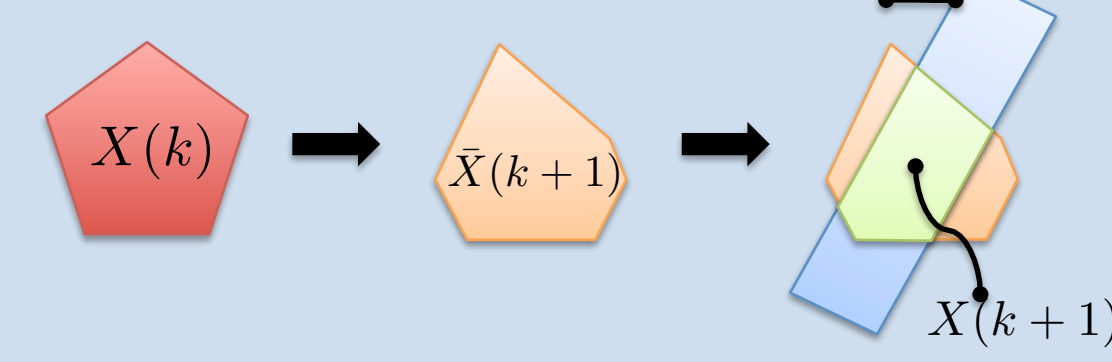
Each node runs a Stochastic Set-Valued Observer (SSVO). The estimates are intersected as to produce less conservative polytopes.

1) Compute the next set-valued estimates

Example of a SVO update for horizon 1.



Example of a SSVO update where low probability events are discarded.



2) Overbound using a hyper-parallelepiped to transmit estimates to neighbors

3) Intersect estimates by performing

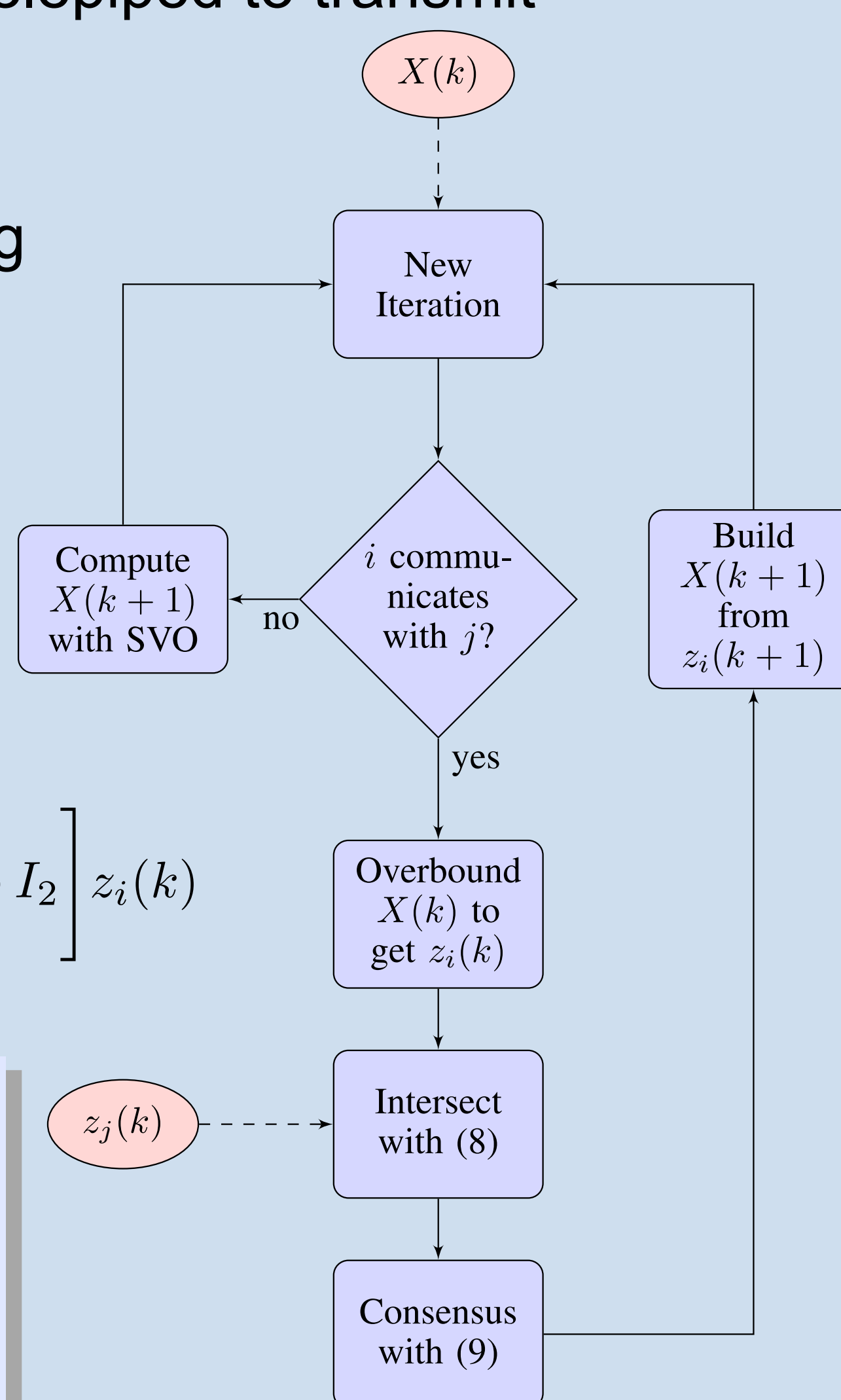
$$z_i(k) = z_j(k) = \max(z_i(k), z_j(k))$$

4) Perform a consensus update on the estimates interval

$$z_i(k+1) = \left[\left(\frac{1}{2} (e_i - e_j)(e_j - e_i)^\top + I_{n_x} \right) \otimes I_2 \right] z_i(k)$$

Remark:

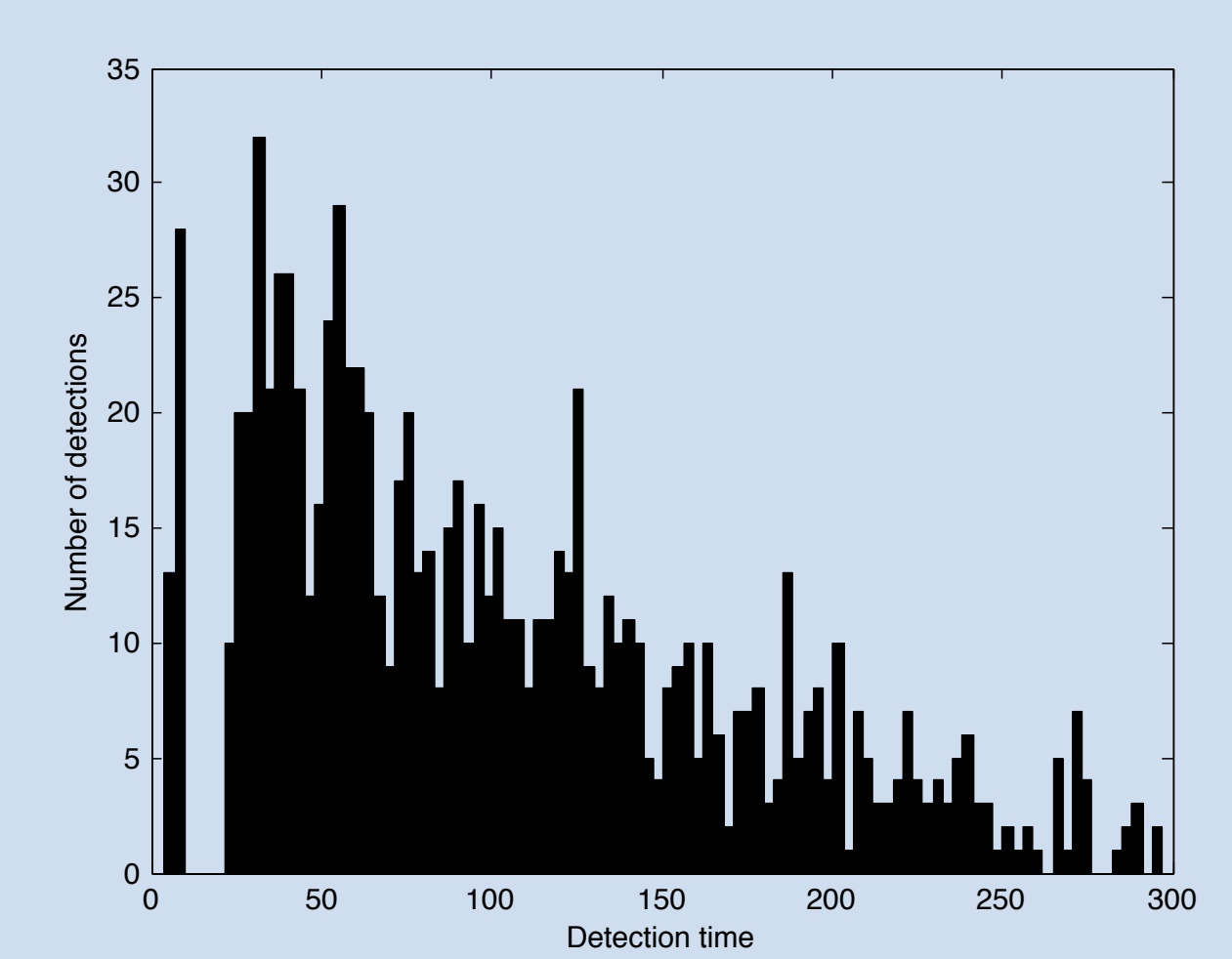
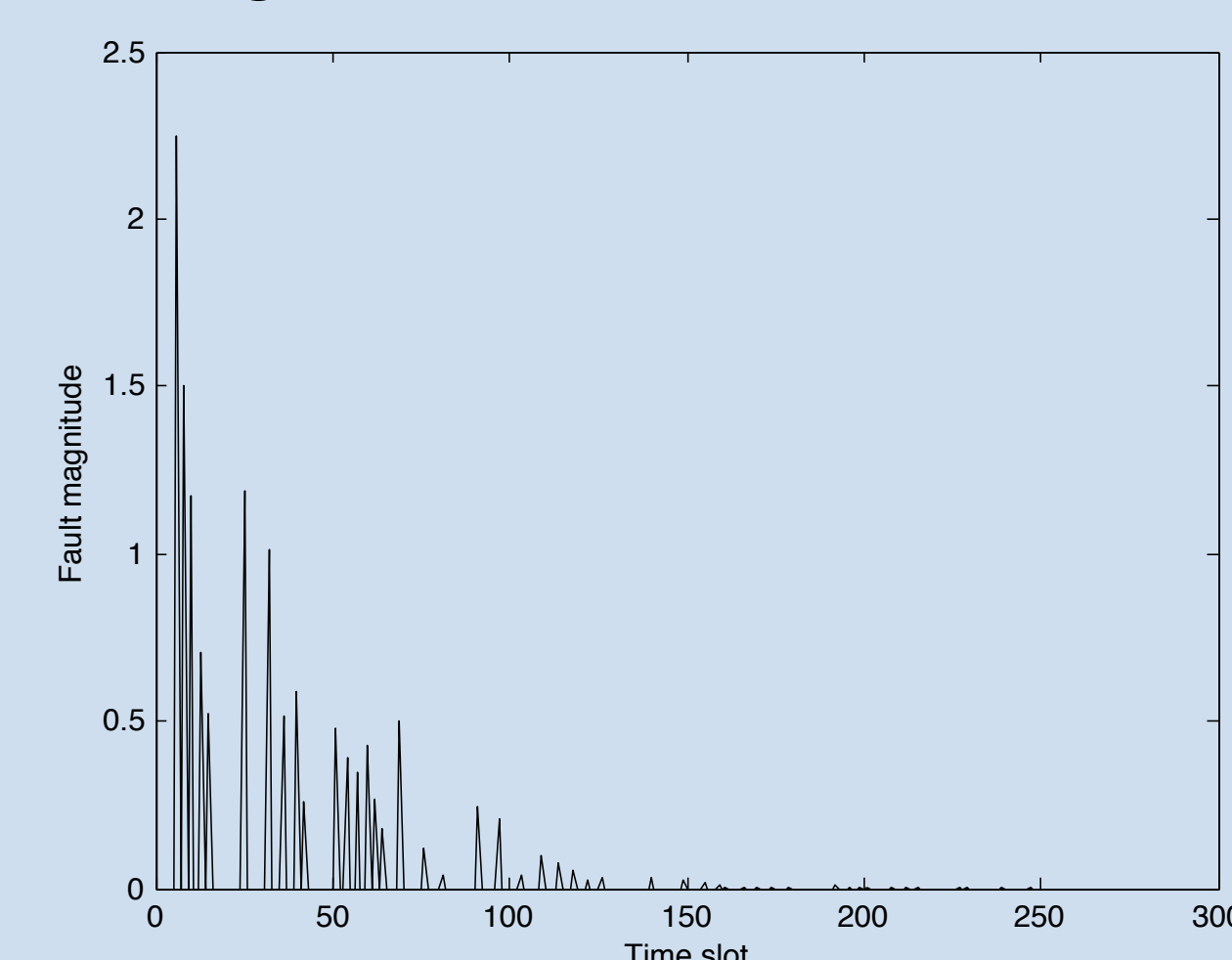
Fault detection consists in checking if the result of the intersection in step 3) is the empty set.



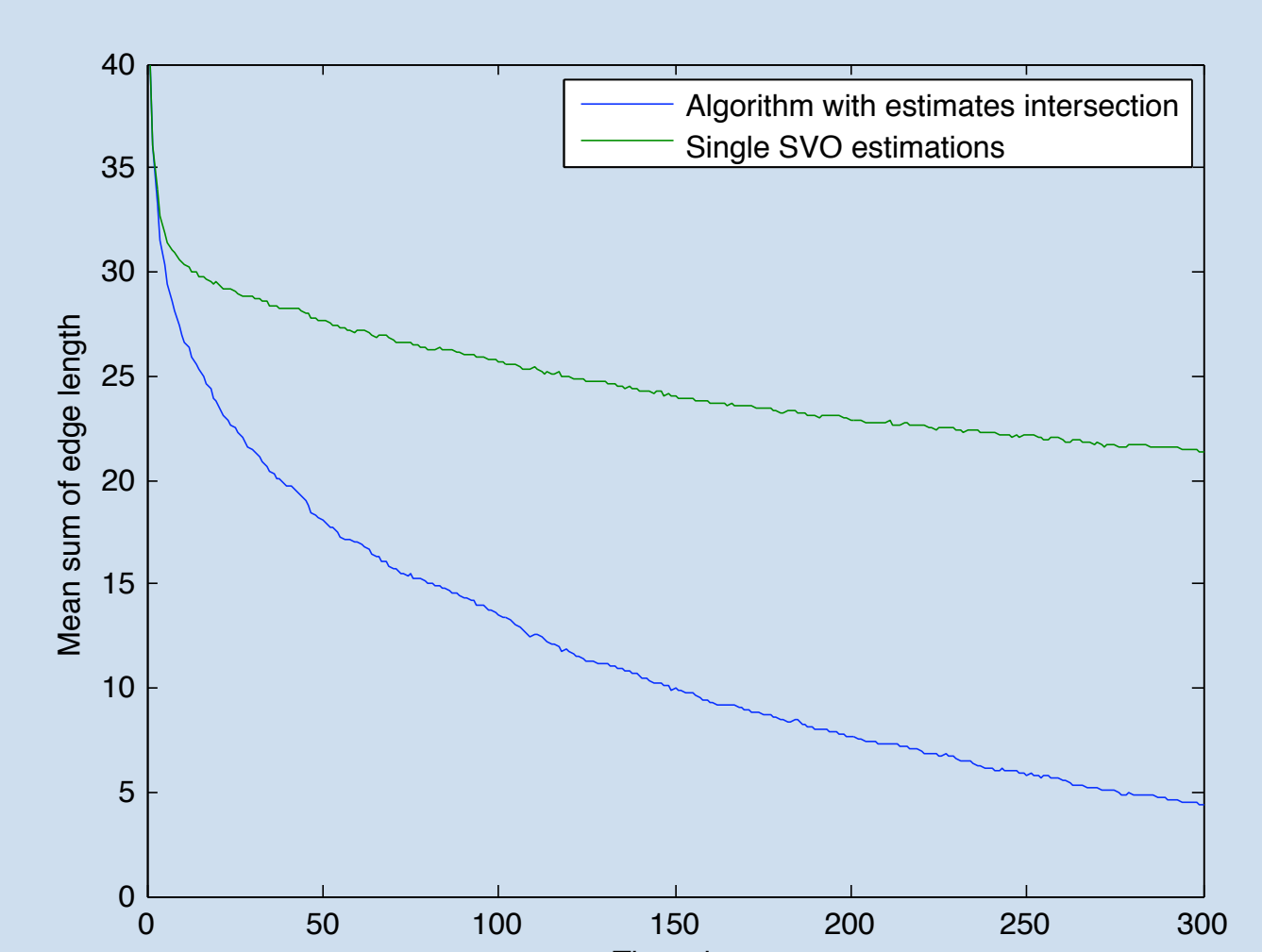
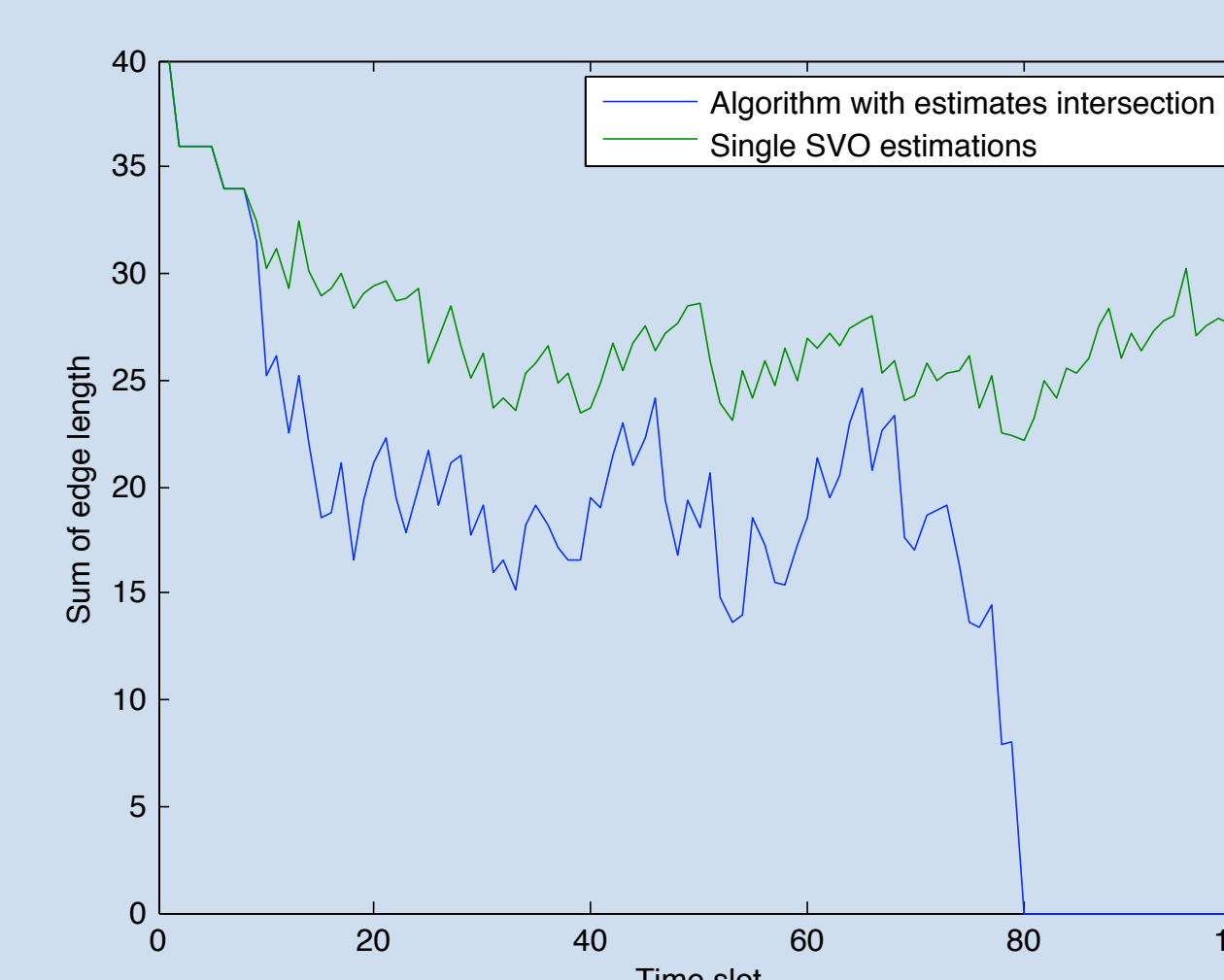
Flowchart of the proposed algorithm

④ Results

- The magnitude of an attacker fault signal is bounded.
- Faults are more likely to be detected as time increases.



- Finite-time consensus is achieved.
- Less conservative set-valued estimates.



Main result:

The algorithm can either detect and intruder or compute the average consensus in finite-time.