Gossip Average Consensus in a Byzantine Environment Using Stochastic Set-Valued Observers

Daniel Silvestre, Paulo Rosa, Rita Cunha, João P. Hespanha, Carlos Silvestre

***Abstract*— We address the problem of a consensus system in the presence of Byzantine faults seen as an attacker injecting a perturbation in the state of the nodes. We propose the use of Set-Valued Observers to detect if the state observations are compatible with the system dynamics. The method is extended to the stochastic case by introducing a strategy to construct a set that is guaranteed to contain all possible states with, at least, a pre-specified desired probability. The proposed algorithm is stable in the sense that it requires a finite number of vertices to represent polytopic sets while also enabling the a priori computation of the largest magnitude of a disturbance that an attacker can inject without being detected.**

I. Introduction

The problem of consensus relates to a set of agents agreeing on a common value using a distributed algorithm. We are interested in randomized gossip average consensus in that nodes are allowed to send messages to a random neighbor in order to compute the average of their initial state. By nature, such algorithms are designed to cope with "crash type" faults by using redundancy and randomization. However, Byzantine faults, such as an intruder in the system, can prevent convergence or drive the system steady state to any value [1]. The applicability of Byzantine fault detection schemes ranges from consensus algorithms to information dissemination and distributed control of industry processes.

The problem of detecting Byzantine faults using unreliable fault detectors was introduced in [2], where multiple classes of theoretic detectors are presented. For the specific case of a consensus system, an algorithm is proposed that makes use of an unreliable detector to solve the problem of consensus. However, [2] differs from our work in that the consensus value is assumed to be one of the initial values, whereas we are looking at asymptotically reaching the average of the initial values by using a linear dynamic system.

The consensus problem has been widely studied when considering rather non-antagonistic failure models which include packet drops and nodes crashing but, to enable a more comprehensive model, Byzantine faults must be considered. The research interest in this issue has motivated a number of contributions including the scenario of unreliable networks in distributed systems. In particular, [1] considers the problem of detecting and correcting the state of the system in the presence of a Byzantine fault. The case of malicious agents and faulty agents is studied and the authors provide, in both cases, bounds on the number of corrupted nodes to ensure detectability of the fault. In [1], the system dynamics are described by a linear time-invariant model that constrains the communication network to be fixed in all time slots. Here, however, a randomized gossip algorithm is considered, thus dropping the assumption that the same sets of nodes are involved in message exchanges at every time instant.

The main contributions of this paper are as follows:

- the analysis of the problem of detecting an intruder in a randomized gossip consensus algorithm is recast into the framework of Linear Parameter-Varying (LPV) systems with uncertain dynamics;
- an upper bound on the magnitude of the attacker signal is derived beyond which the attacker can be detected;
- the concept of stochastic Set-Valued Observers (SVOs) is introduced by taking advantage of the use of α-confidence sets, i.e., sets where the state of the system is known to belong with a desired pre-specified $1-\alpha$ probability; which can be viewed as a generalization of confidence intervals;
- finally, it is also shown that this method inherits the main properties regarding the computational stability by having a bound on the volume and number of vertices needed to define the polytopic sets where the state is contained with a pre-specified probability; guarantees of intruder detection are also provided.

The choice for representing the set of possible states depends on a mathematical formulation that enables fast and non-conservative intersections and unions of sets, as those are major and normally time-consuming operations when implemented in a computer. One alternative is to use the concept of zonotopes, described in [3] and further developed in [4] and [5]. In this article, an alternative approach is adopted, based on the concept of Set-Valued Observers (SVOs) first introduced in [6] and [7] and further information can be found in [8] and [9] and the references therein.

The applicability of the proposed method for fault detection in a randomized gossip algorithm is not limited to the consensus problem, as several challenges in the Fault Detection and Isolation (FDI) literature - [10], [11] - share the framework described in the sequel. In [12], the authors propose the use of SVOs for fault detection as a model falsification problem. This paper extends the results in [12] to detect Byzantine faults in consensus systems, by rewriting its dynamics as an LPV. Moreover, unlike the approach in [12], the method proposed herein takes into account the probability information related to having a given communication.

Contact the author D. Silvestre through `dsilvestre@isr.ist.utl.pt` for a free unformatted copy of this article or go to http://ieeexplore.ieee.org/abstract/document/6760562/ for the final formatted version.

References

[1] F. Pasqualetti, A. Bicchi, and F. Bullo, "Consensus computation in unreliable networks: A system theoretic approach," *Automatic Control, IEEE Transactions on*, vol. 57, no. 1, pp. 90 –104, jan. 2012.

[2] K. P. Kihlstrom, L. E. Moser, and P. M. Melliar-Smith, "Solving consensus in a byzantine environment using an unreliable fault detector," in *Proceedings of the International Conference on Principles of Distributed Systems (OPODIS)*, 1997, pp. 61–75.

[3] D. Bertsekas and I. Rhodes, "Recursive state estimation for a set-membership description of uncertainty," *Automatic Control, IEEE Transactions on*, vol. 16, no. 2, pp. 117 – 128, apr 1971.

[4] C. Combastel, "A state bounding observer for uncertain non-linear continuous-time systems based on zonotopes," in *Decision and Control, 2005 and 2005 European Control Conference. CDC-ECC '05. 44th IEEE Conference on*, dec. 2005, pp. 7228 – 7234.
[5] T. Alamo, J. Bravo, and E. Camacho, "Guaranteed state estimation by zonotopes," *Automatica*, vol. 41, no. 6, pp. 1035 – 1043, 2005.
[6] H. Witsenhausen, "Sets of possible states of linear systems given perturbed observations," *Automatic Control, IEEE Transactions on*, vol. 13, no. 5, pp. 556 – 558, oct 1968.
[7] F. Schweppe, "Recursive state estimation: Unknown but bounded errors and system inputs," *Automatic Control, IEEE Transactions on*, vol. 13, no. 1, pp. 22 – 28, feb 1968.
[8] F. Schweppe., *Uncertain Dynamic Systems*. Prentice-Hall, 1973.
[9] M. Milanese and A. Vicino, "Optimal estimation theory for dynamic systems with set membership uncertainty: An overview," *Automatica*, vol. 27, no. 6, pp. 997 – 1009, 1991.
[10] R. J. Patton, "Fault-tolerant control systems: The 1997 situation," in *IFAC symposium on fault detection supervision and safety for technical processes*, vol. 3, 1997.
[11] J. Bokor and Z. Szabó, "Fault detection and isolation in nonlinear systems," in *Annual Reviews in Control 33.2*, 2009, pp. 113–123.
[12] P. Rosa, C. Silvestre, J. Shamma, and M. Athans, "Fault detection and isolation of LTV systems using set-valued observers," in *Proceedings of the 49th IEEE Conference on Decision and Control*, December 2010, pp. 768–773.