Asymmetric Randomized Gossip Algorithm for Average Consensus

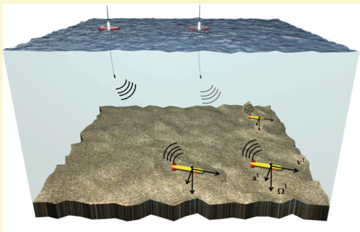
Daniel Silvestre, Duarte Antunes, Carlos Silvestre

{dsilvestre,dantunes,cjs}@isr.ist.utl.pt

Objectives:
- Compute the average of the initial states using asymmetric and randomized communications
- Guarantee almost sure convergence, convergence in mean square sense and convergence in expected value.

1. Motivation

The problem appears in many scenarios where nodes need to converge to the same point or agree on a common value.



2. Problem statement

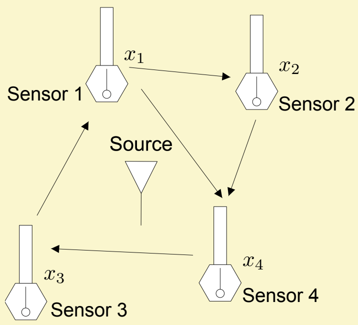
The objective is to compute the average of some quantity of interest. In the network below, the sensors aim to estimate the temperature by taking the average of the measurements.

Network Model

$$G = (V, E)$$
$$E \subseteq V \times V$$

System Model

$$x(t_{k+1}) = U_k x(t_k)$$
$$U_k = Q_{ij}, \text{ with probability } w_{ij}$$



Objective:

$$\lim_{t\to\infty} x_i(t) = \frac{1}{m}\sum_{i=1}^{m} x_i(0)$$

Convergence Definitions

(i)*Converges almost surely to average consensus* if

$$\lim_{t\to\infty} x_i(t) = \frac{1}{m}\sum_{i=1}^{m} x_i(0) \quad , \quad \forall_{i\in\{1,\dots,m\}} \quad \text{almost surely.}$$

(ii) *Converges in Expectation to average consensus* if

$$\lim_{t\to\infty} \mathbb{E}[x_i(t)] = x_{\text{av}} \quad , \quad \forall_{i\in\{1,\dots,m\}}.$$

(iii) *Converges in Second Moment to average consensus* if

$$\lim_{t\to\infty} \mathbb{E}[(x_i(t) - x_{\text{av}})^2] \to 0 \quad , \quad \forall_{i\in\{1,\dots,m\}}.$$

3. Proposed Solution

The proposed solution uses the concept of keeping an auxiliary variable to maintain the state sum.

The parameters α, β, γ are design choices to optimize the speed of convergence and depend on the network topology.

$$Q_{ij} = \begin{bmatrix} A_{ij} & B_{ij} \\ C_{ij} & D_{ij} \end{bmatrix}$$

$$\Lambda_i := \text{diag}(\mathsf{e}_i)$$

$$\Omega_{ij} := I - (\Lambda_i + \Lambda_j)$$

$$A_{ij} := I - \alpha\Lambda_j + \alpha \mathsf{e}_j \mathsf{e}_i^\intercal$$
$$B_{ij} := \beta\Lambda_j + \gamma \mathsf{e}_j \mathsf{e}_i^\intercal$$
$$C_{ij} := \Lambda_j(I - A_{ij})$$
$$D_{ij} := \Omega_{ij} + \Lambda_j(I + \mathsf{e}_j \mathsf{e}_i^\intercal - B_{ij}).$$

4. Results

- The optimization of the convergence rate is possible using a Semidefinite Program even though the problem is non-convex.
- The algorithm converges in Expectation.
- In the worst case, the ratio of the convergence time between the asymmetric and the symmetric case is related by a constant.
- The performance of the asymmetric algorithm is better in the worst case network scenario than the symmetric case but for the best case network scenario it underperforms.

	Lower bound		Upper bound	
Case	b_ticks	u_ticks	b_ticks	u_ticks
Best	8	25.67	48.02	154.01
Worst	59.12	116.93	354.71	701.56

- Convergence rates for the second moment and expectation are computed in discrete and continuous time.

Main result:

Convergence in Mean Square Sense is proven using a Lyapunov Function and assuming a degree of connectivity in the network.

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